8.1 Absolute Value Functions

We will explore one final function family this year known as piecewise functions. **Piecewise functions** are functions that are defined a piece at a time. In other words, for one section of the domain, the function will look one way, and in another part of the domain, the function will look different. The simplest example of a piecewise function is the absolute value function.

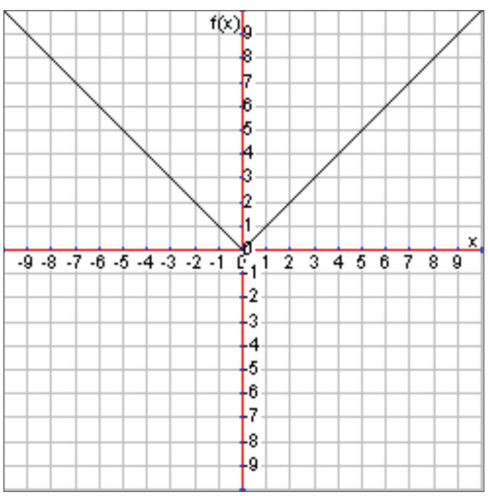
Overview of Absolute Value Functions

The absolute value is a measure of how far away from zero something is. For example, the absolute value of -7 is just 7 because it is seven units away from zero. We use bars on either side of a number or variable to show absolute value like this: $|-7| = 7$.

We can do the same thing with functions. For example, the parent function for absolute value functions is $f(x) = |x|$. This means when we input a value for the variable, the output is the distance from the input to zero. With this knowledge in hand, we can evaluate and graph.

Evaluating and Graphing Absolute Value Functions

Let's start by evaluating and graphing the parent function, $f(x) = |x|$. We'll make an x/y chart and plot the points on the coordinate plane as follows:



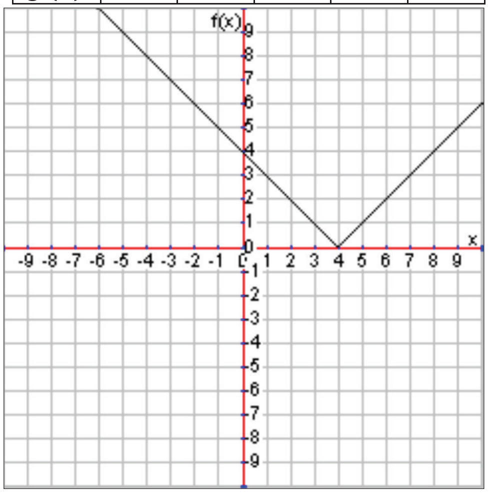
x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2

Notice that the graph looks like a "V". It's not a parabola, but it appears to have something like vertex. The reason it does this is because all the negative values become positive through the absolute value function.

Let's look at two more examples that will lead us into thinking about transformations of absolute value functions. We'll make an x/y chart and graph $g(x) = |x - 4|$ and $h(x) = |x - 4| - 7$.

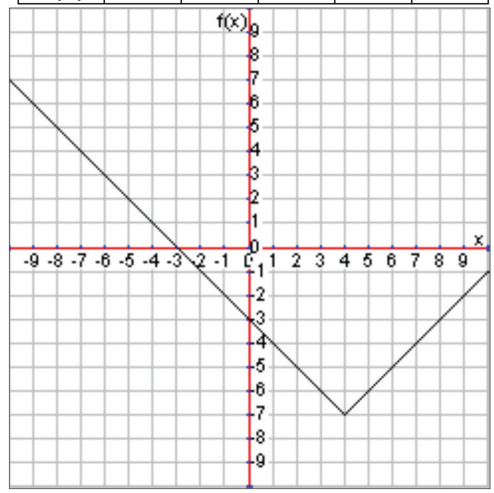
$g(x) = |x - 4|$

x	2	3	4	5	6
$g(x)$	2	1	0	1	2



$h(x) = |x - 4| - 7$

x	2	3	4	5	6
$h(x)$	-5	-6	-7	-6	-5



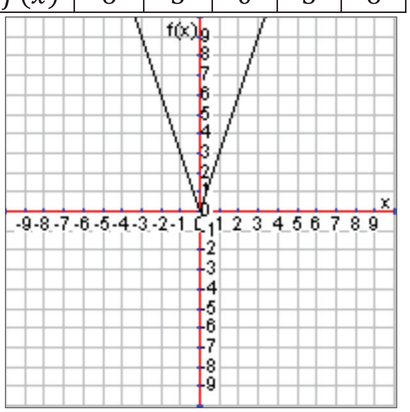
Notice that the absolute value can be translated (or shifted) around even leading to times when there are negative outputs. So how do these transformations work?

Transforming Absolute Value Functions

The general form of a linear absolute value function is $f(x) = a|x - h| + k$. As you might already be able to guess, the h value controls the shift left and right while the k value controls the shift up and down. If you had to take a stab at it, you would probably say that the a value controls the tilt or slope of the line. You would be correct. Let's take a look at a few more transformations.

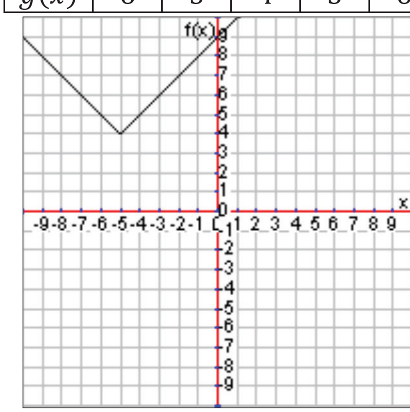
$f(x) = 3|x|$

x	-2	-1	0	1	2
$f(x)$	6	3	0	3	6



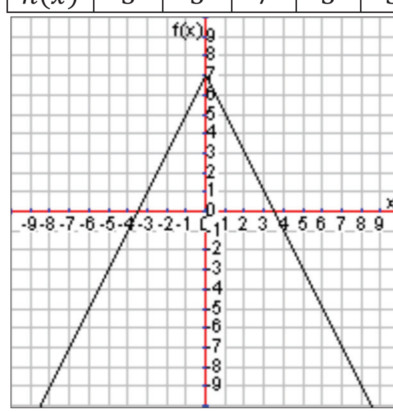
$g(x) = |x + 5| + 4$

x	-3	-4	-5	-6	-7
$g(x)$	6	5	4	5	6



$h(x) = -2|x| + 7$

x	-2	-1	0	1	2
$h(x)$	3	5	7	5	3



All of these transformations should be about what you would expect at this point in the course. The interesting one is the negative a value in $h(x)$ which reflected the graph so that we get a mountain top instead of the "V" shape of the pointed down.

Solving Absolute Value Equations

To solve absolute value equations, we could graph the function and look for the input where the height is what we want. However, by now you should know that we'll want an algebraic method for solving as well. The key to the algebraic method is realizing that there are two ways that we can get any given value. For example, if we have the function $f(x) = |x|$ and want to solve for $|x| = 7$, the two answers would be $x = 7$ and $x = -7$. When the inside of the absolute value equals exactly what we want or when it is the opposite (the negative) of what we want, we get our two solutions.

All this means that when we solve ABS (shorthand for absolute value since I'm tired of typing it so many times) equations, we need to consider both the case when inside the ABS symbol is positive and the case when the inside of the ABS symbol is negative. Let's take a slightly more complicated example:

$$|x - 7| = 5$$

Positive Case

$$x - 7 = 5$$
$$+7 \quad +7$$
$$x = 12$$

Negative Case

$$-(x - 7) = 5$$
$$-x + 7 = 5$$
$$-7 \quad -7$$
$$-x = -2$$
$$\frac{-x}{-1} = \frac{-2}{-1}$$
$$x = 2$$

So we get two answers. $|x - 7| = 5$ when $x = 12$ and when $x = 2$. Notice that we had to take the opposite of everything in the ABS symbol meaning we had to distribute the negative sign.

Solving Absolute Value Inequalities

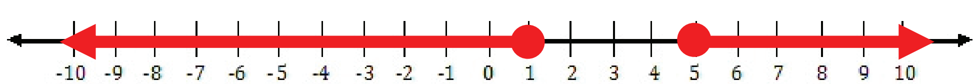
ABS inequalities work the same way except that our solution will be plotted on the number. We'll still need to consider both the positive case and the negative case. Solving the problem $|x - 3| \geq 2$ gives us the following solution:

Positive Case

$$x - 3 \geq 2$$
$$+3 \quad +3$$
$$x \geq 5$$

Negative Case

$$-(x - 3) \geq 2$$
$$-x + 3 \geq 2$$
$$-3 \quad -3$$
$$-x \geq -1$$
$$\frac{-x}{-1} \leq \frac{-1}{-1}$$
$$x \leq 1$$



Plotting on the number line gives us our final solution. You can double check by plugging in any input in the shaded area of the number line. For example, inputting $x = 0$ makes the statement $|x - 3| \geq 2$ true, but inputting $x = 3$ makes the statement false.

Real-Life Absolute Value

Often times we talk about margin of error. For example, on the MAP test there is a margin of error of three. This means that if you score a 234 on the MAP test and retook the test, you should score within three points of 234 on the next test. In other words, you would score anywhere from 231 to 237 should you retake the MAP test. We can model situations like this using absolute value.

Let $r =$ score on your MAP retake assuming you got a score of 234 on the original test. Then our absolute value inequality would be $|r - 234| \leq 3$. We can again set this up with two cases to solve.

Positive Case

$$r - 234 \leq 3$$
$$+234 \quad +234$$
$$r \leq 237$$

Negative Case

$$-(r - 234) \leq 3$$
$$-r + 234 \leq 3$$
$$-234 \quad -234$$
$$-r \leq -231$$
$$\frac{-r}{-1} \geq \frac{-231}{-1}$$
$$r \geq 231$$

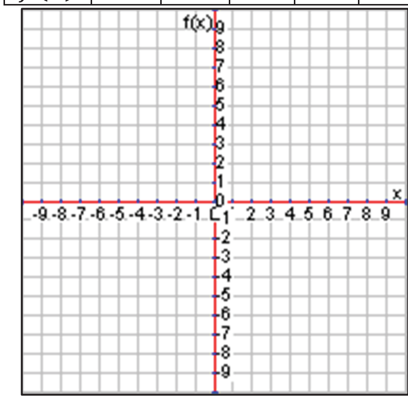
So again, we see that the score on the retake should be greater than or equal to 231 but less than or equal to 237. Other situations where we use the margin of error include manufacturing where measurements are taken or partial credit on tests where you can be within a given amount of the actual answer for your estimate to count.

Lesson 8.1

Fill out an x/y chart and graph each of the following functions.

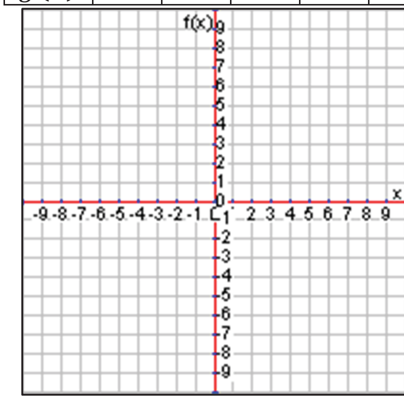
1. $f(x) = \frac{1}{2}|x|$

x					
$f(x)$					



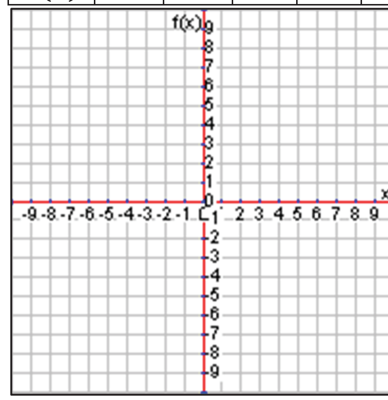
2. $g(x) = |x + 2|$

x					
$g(x)$					



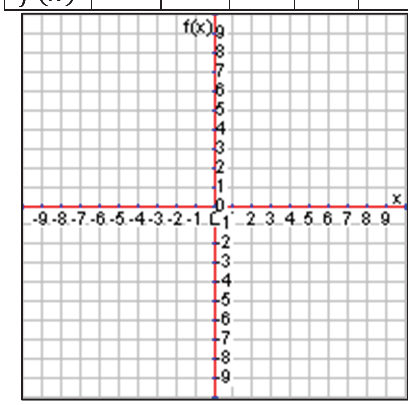
3. $h(x) = -|x| - 4$

x					
$h(x)$					



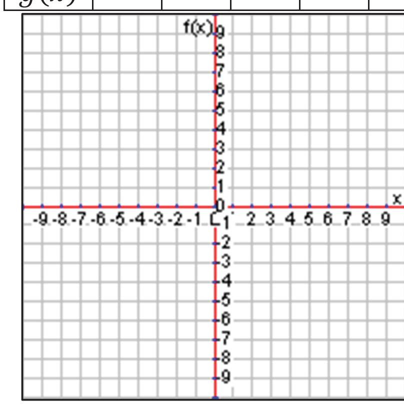
4. $f(x) = |x - 4| + 5$

x					
$f(x)$					



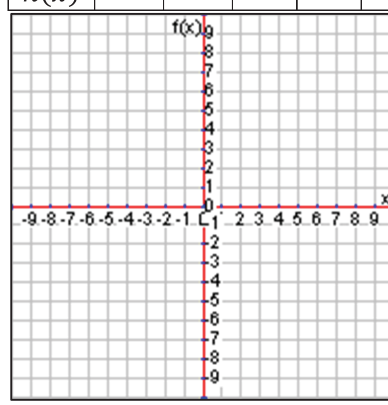
5. $g(x) = 2|x| - 4$

x					
$g(x)$					



6. $h(x) = -\frac{1}{3}|x - 1|$

x					
$h(x)$					



Given that $f(x) = |\mathbf{x}|$ is the parent function, describe the transformation of $g(x)$.

7. $g(x) = |x - 3| + 2$

8. $g(x) = -|x + 4|$

9. $g(x) = 2|x| - 6$

10. $g(x) = |x - 2| + 7$

11. $g(x) = -\frac{1}{2}|x|$

12. $g(x) = \frac{1}{3}|x - 1| + 5$

Solve the following equations.

13. $|x-3|=6$

14. $2|x|+5=9$

15. $|x+5|+2=7$

16. $|x+2|-10=-4$

17. $2|x-7|+3=15$

18. $-\frac{1}{2}|x|+4=24$

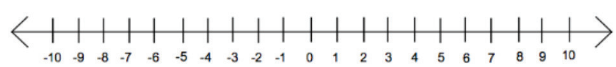
Solve the following inequalities.

19. $|x+2|>4$

20. $|x-4|-1\leq 0$

21. $2|x|-3\leq 3$

22. $\frac{1}{2}|x+5|>1$



23. $-2|x+3|+5>1$

24. $|x-6|+2\leq 3$

Set-up an absolute value inequality to model each of the following situations.

25. On the MAP test you originally scored a 272. To validate your high test score, the school wants you to retake the test. According to the MAP test guidelines, you should score within 3 points of your original score on a retake.

26. A factory produces cereal boxes that are supposed to be 12 inches high, but there is an acceptable margin of error of 0.1 inch.

27. The correct answer to a system of linear equations is the point $(2.8, -3.2)$, but the teacher says your estimate can be within half a unit for both coordinates. *(Note: You need to set-up two absolute value inequalities - one for the x-coordinate and one for the y-coordinate.)*

28. A couple invites 300 people to their wedding knowing that all of them may not show-up and that some people invited may bring along extra people. To plan for such an event, they decide to include of margin of error of 20 people when they talk to the caterer.