NAME ______________________________ DATE ______________ PERIOD _________

3-5

Study Guide

Continuity and End Behavior

A function is **continuous** at $x = c$ if it satisfies the following three conditions.
(1) the function is defined at c; in other words, $f(c)$ exists;
(2) the function approaches the same y-value to the left and right of $x = c$; and
(3) the y-value that the function approaches from each side is $f(c)$.

Functions can be continuous or **discontinuous.** Graphs that are discontinuous can exhibit **infinite discontinuity**, **jump discontinuity**, or **point discontinuity.**

Example 1 **Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.**

a. $f(x) = 2|x| + 3$; $x = 2$

(1) The function is defined at $x = 2$; $f(2) = 7$.

(2) The tables below show that y approaches 7 as x approaches 2 from the left and that y approaches 7 as x approaches 2 from the right.

x	$y = f(x)$
1.9	6.8
1.99	6.98
1.999	6.998

x	$y = f(x)$
2.1	7.2
2.01	7.02
2.001	7.002

(3) Since the y-values approach 7 as x approaches 2 from both sides and $f(2) = 7$, the function is continuous at $x = 2$.

b. $f(x) = \frac{2x}{x^2 - 1}$; $x = 1$

Start with the first condition in the continuity test. The function is not defined at $x = 1$ because substituting 1 for x results in a denominator of zero. So the function is discontinuous at $x = 1$.

c. $f(x) = \begin{cases} 2x + 1 & \text{if } x > 2 \\ x - 1 & \text{if } x = 2 \end{cases}$

This function fails the second part of the continuity test because the values of $f(x)$ approach 1 as x approaches 2 from the left, but the values of $f(x)$ approach 5 as x approaches 2 from the right.

The **end behavior** of a function describes what the y−values do as $|x|$ becomes greater and greater. In general, the end behavior of any polynomial function can be modeled by the function made up solely of the term with the highest power of x and its coefficient.

Example 2 **Describe the end behavior of $p(x) = -x^5 + 2x^3 - 4$.**

Determine $f(x) = a_n x^n$ where x^n is the term in $p(x)$ with the highest power of x and a_n is its coefficient.

$f(x) = -x^5 \quad x^n = x^5 \quad a_n = -1$

Thus, by using the table on page 163 of your text, you can see that when a^n is negative and n is odd, the end behavior can be stated as $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to \infty$ as $x \to -\infty$.

NAME ______________________ DATE ____________ PERIOD ________

3-5 Practice

Continuity and End Behavior

Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.

1. $y = \frac{2}{3x^2}$; $x = -1$

2. $y = \frac{x^2 + x + 4}{2}$; $x = 1$

3. $y = x^3 - 2x + 2$; $x = 1$

4. $y = \frac{x - 2}{x + 4}$; $x = -4$

Describe the end behavior of each function.

5. $y = 2x^5 - 4x$

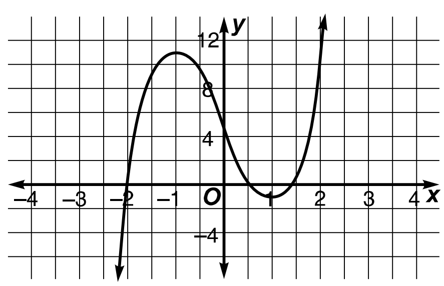
6. $y = -2x^6 + 4x^4 - 2x + 1$

7. $y = x^4 - 2x^3 + x$

8. $y = -4x^3 + 5$

Given the graph of the function, determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing.

9.



10. ***Electronics*** Ohm's Law gives the relationship between resistance R, voltage E, and current I in a circuit as $R = \frac{E}{I}$. If the voltage remains constant but the current keeps increasing in the circuit, what happens to the resistance?