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3-2 Study Guide and Intervention

Logarithmic Functions

Logarithmic Functions and Expressions The inverse relationship between logarithmic functions and exponential functions can be used to evaluate logarithmic expressions.

If $b > 0$, $b \neq 1$, and $x > 0$, then

Logarithmic Form		Exponential Form
$\log_b x = y$	if and only if	$b^y = x$

base: b (in $\log_b$ and in b^y); exponent: y

The following properties are also useful.

$\log_b 1 = 0$ $\quad \log_b b = 1$ $\quad \log_b b^x = x$ $\quad b^{\log_b x} = x,\ x > 0$

Example 1 **Evaluate each logarithm.**

a. $\log_5 \frac{1}{25}$

$\log_5 \frac{1}{25} = y$ Let $\log_5 \frac{1}{25} = y$.

$5^y = \frac{1}{25}$ Write in exponential form.

$5^y = 5^{-2}$ $\frac{1}{25} = 5^{-2}$

$y = -2$ Equality Prop. of Exponents

Therefore, $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

b. $\log_3 \sqrt{3}$

$\log_3 \sqrt{3} = y$ Let $\log_3 \sqrt{3} = y$.

$3^y = \sqrt{3}$ Write in exponential form.

$3^y = 3^{\frac{1}{2}}$ $3^{\frac{1}{2}} = \sqrt{3}$

$y = \frac{1}{2}$ Equality Prop. of Exponents

Therefore, $\log_3 \sqrt{3} = \frac{1}{2}$ because $3^{\frac{1}{2}} = \sqrt{3}$.

Example 2 **Evaluate each expression.**

a. $\ln e^7$

$\ln e^7 = 7$ $\ln e^x = x$

b. $e^{\ln 5}$

$e^{\ln 5} = 5$ $e^{\ln x} = x$

c. $10^{\log 13}$

$10^{\log 13} = 13$ $10^{\log x} = x$

Exercises

Evaluate each logarithm.

1. $\log_7 7$
2. $10^{\log 5x}$
3. $3^{\log_3 2}$
4. $\log_6 36$
5. $\log_3 \frac{1}{81}$
6. $e^{\ln x^2}$

7. **FINANCIAL LITERACY** Ms. Dasilva has \$3000 to invest. She would like to invest in an account that compounds continuously at 6%. Use the formula $\ln A - \ln P = rt$, where A is the current balance, P is the original principal, r is the rate as a decimal, and t is the time in years. How long will it take for her balance to be \$6000?

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3-2 Practice

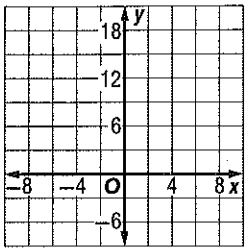
Logarithmic Functions

Evaluate each expression.

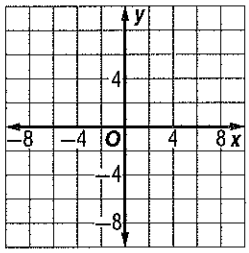
1. $\log_7 7^3$
2. $\log_{10} 0.001$
3. $\log_8 4096$
4. $2 \ln e^5$
5. $9^{\log_9 18}$
6. $\log_8 32$
7. $\log_6 216$
8. $e^{\ln 0.014x}$
9. $\log_{12} 144$

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

10. $g(x) = 4^{-x+2}$

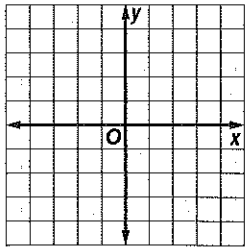


11. $g(x) = e^{2x+1}$



12. Use the graph of f to describe the transformation that results in the graph of g. Then sketch the graphs of f and g.

$f(x) = \ln x, g(x) = \ln\left(\frac{x}{2}\right) - 2$



13. **INVESTMENTS** The annual growth rate r for an investment can be found using $r = \frac{1}{t} \ln \frac{P}{P_0}$, where t is time in years, P is the present value, and P_0 is the original investment. An investment of \$4000 was made in 2005 and had a value of \$7500 in 2010. What was the average growth rate of the investment?

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3-3 Study Guide and Intervention *(continued)*

Properties of Logarithms

Change of Base Formula If the logarithm is in a base that needs to be changed to a different base, the **Change of Base Formula** is required.

For any positive real numbers a, b, and x, $a \neq 1$, $b \neq 1$, $\log_b x = \dfrac{\log_a x}{\log_a b}$.

Many non-graphing calculators cannot be used for logarithms that are not base e or base 10. Therefore, you will often use this formula, especially for scientific applications. Either of the following forms will provide the correct answer.

$$\log_b x = \frac{\log x}{\log b} \qquad \log_b x = \frac{\ln x}{\ln b}$$

Example **Evaluate each logarithm.**

a. $\log_2 7$

$\log_2 7 = \dfrac{\ln 7}{\ln 2}$ Change of Base Formula

≈ 2.81 Use a calculator.

b. $\log_{\frac{1}{3}} 10$

$\log_{\frac{1}{3}} 10 = \dfrac{\log 10}{\log \frac{1}{3}}$ Change of Base Formula

≈ -2.10 Use a calculator.

Exercises

Evaluate each logarithm.

1. $\log_{32} 631$

2. $\log_3 17$

3. $\log_7 1094$

4. $\log_6 94$

5. $\log_5 256$

6. $\log_9 712$

7. $\log_6 832$

8. $\log_{11} 47$

9. $\log_3 9$

10. $\log_8 256$

11. $\log_{12} 4302$

12. $\log_{0.5} 420$

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3-3 Practice

Properties of Logarithms

Express each logarithm in terms of ln 10 and ln 3.

1. $\ln 300$

2. $\ln 27000$

3. $\ln \frac{10}{9}$

4. $\ln \frac{729}{10000}$

Expand each expression.

5. $\ln \frac{x+1}{\sqrt[4]{x-5}}$

6. $\ln \frac{x^2}{\sqrt{3x+2}}$

7. $\log_2 [(2x)^3(x+1)]$

8. $\log_8 [(4x+2)^3(x-4)]$

9. $\log_{13} \frac{3x^4}{\sqrt[3]{7x-3}}$

10. $\log_2 \frac{(x+1)^3}{\sqrt[3]{x+5}}$

Condense each expression.

11. $\frac{1}{2} \ln (3x - 5y) - \ln (4x + y)$

12. $3 \log_2 (5x + 6) - \frac{1}{2} \log_2 (x - 4)$

13. $2 - \log_7 6 - 2 \log_7 x$

14. $\log_3 8 + \log_3 x - 2 \log_3 (x + 4)$

15. $\log y + \log 3 - \frac{1}{3} \log(x) + 2 \log z$

16. $\log_3 y + \log_3 x - \frac{1}{2} \log_3 x + 3 \log_3 z$

Evaluate each logarithm.

17. $\log_{\frac{1}{2}} \frac{1}{5}$

18. $\log_{100} 200$

19. $\log_{0.01} 4$

20. $\log_{0.24} 322$

21. $\log_6 24$

22. $\log_{\frac{1}{3}} 9.8$

23. SEISMOLOGY The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where R is the magnitude, I is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

a. Guam region, in 2008, $R = 6.7$

b. Macquarie Island region, in 2008, $R = 7.1$