Interactive Classroom

Glencoe McGraw-Hill

Algebra 2

LESSON 7-3

Square Root Functions and Inequalities

Click the mouse button
or press the space bar to continue.

New Vocabulary

- square root function
- radical function
- square root inequality

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Key Concept
Parent Function of Square Root Functions

For Your FOLDABLE

Parent function: $f(x) = \sqrt{x}$

Domain: $\{x \mid x \geq 0\}$

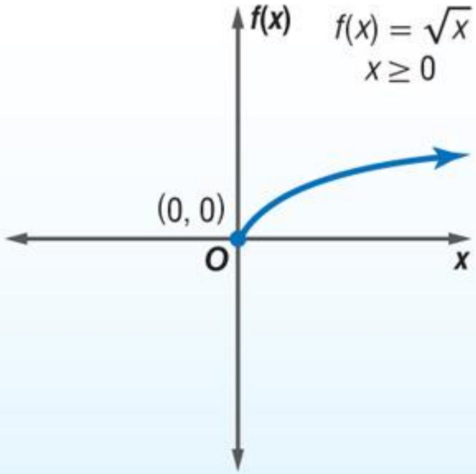
Range: $\{y \mid y \geq 0\}$

Intercepts: $x = 0, y = 0$

Not defined: $x < 0$

End behavior: $x \to 0, y \to 0$

$x \to +\infty, y \to +\infty$



EXAMPLE 1 Identify Domain and Range

Identify the domain and range of $f(x) = \sqrt{x-2}$.

The domain only includes values for which the radicand is nonnegative.

$x - 2 \geq 0$	Write an inequality.
$x \geq 2$	Add 2 to each side.

Thus, the domain is $\{x \mid x \geq 2\}$.

Find $f(2)$ to determine the lower limit of the range.

$f(2) = \sqrt{2-2}$ or 0

So, the range is $\{y \mid y \geq 0\}$.

Answer: D: $\{x \mid x \geq 2\}$; R: $\{y \mid y \geq 0\}$

EXAMPLE 1 Check Your Progress

CheckPoint

Identify the domain and range of $f(x) = 3\sqrt{x+4}$.

A. D: $\{x \mid x \geq -4\}$; R: $\{y \mid y \leq 0\}$

B. D: $\{x \mid x \geq 4\}$; R: $\{y \mid y \geq 0\}$

C. D: $\{x \mid x \geq -4\}$; R: $\{y \mid y \geq 0\}$

D. D: $\{x \mid \text{all real numbers}\}$; R: $\{y \mid y \geq 0\}$

Key Concept Transformations of Square Root Functions

For Your FOLDABLE

$$f(x) = a\sqrt{x - h} + k$$

h—Horizontal Translation	*k*—Vertical Translation
$\lvert h \rvert$ units right if h is positive $\lvert h \rvert$ units left if h is negative	$\lvert k \rvert$ units up if k is positive $\lvert k \rvert$ units down if k is negative
The domain is $\{x \mid x \geq h\}$.	The range is $\{y \mid y \geq k\}$.

a—Orientation and Shape

- If $a < 0$, the graph is reflected across the x-axis.
- If $\lvert a \rvert > 1$, the graph is vertically expanded.
- If $0 < \lvert a \rvert < 1$, the graph is vertically compressed.

EXAMPLE 2 Graph Square Root Functions

A. Graph the function $y = 3\sqrt{x-4} + 2$. State the domain and range.

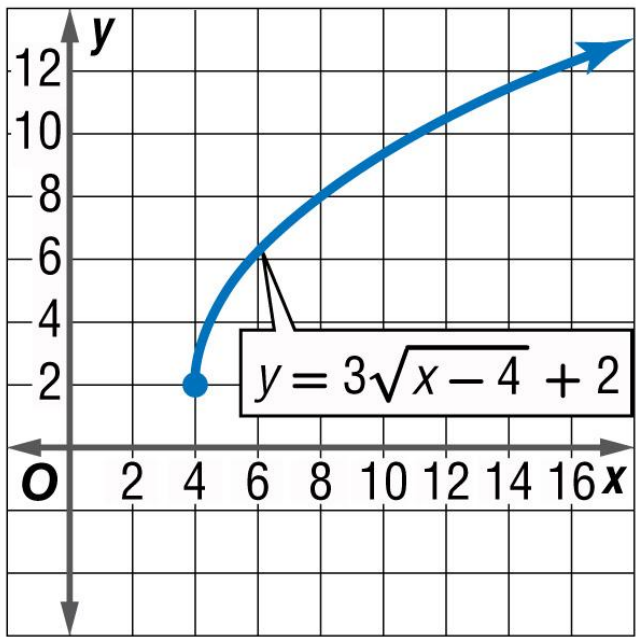
The minimum point is at $(h, k) = (4, 2)$. Make a table of values for $x \geq 4$ and graph the function. The graph is the same shape as $f(x) = \sqrt{x}$, but because $|a| \geq 1$ the graph is vertically compressed. It is also translated 4 units right and 2 units up.

EXAMPLE 2 Graph Square Root Functions

Notice the end behavior; as x increases, y increases.

Answer: The domain is $\{x \mid x \geq 4\}$ and the range is $\{y \mid y \geq 2\}$.

x	y
4	2
5	5
6	6.2
7	7.2
8	8

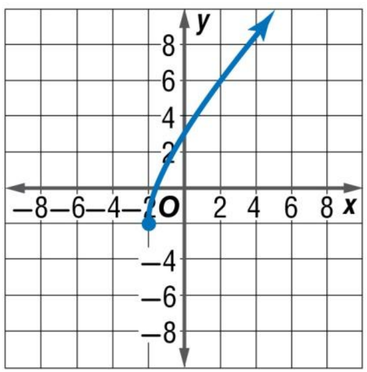


EXAMPLE 2 Check Your Progress

CheckPoint

A. Graph the function $y = 3\sqrt{x - 2} - 3$.

A.



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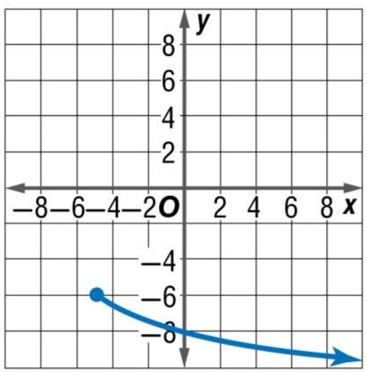

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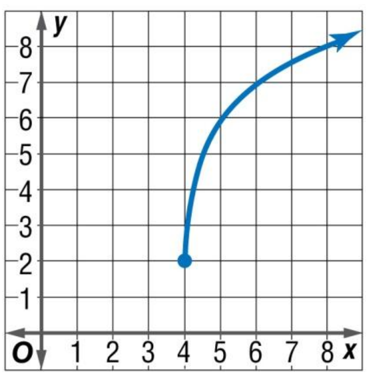

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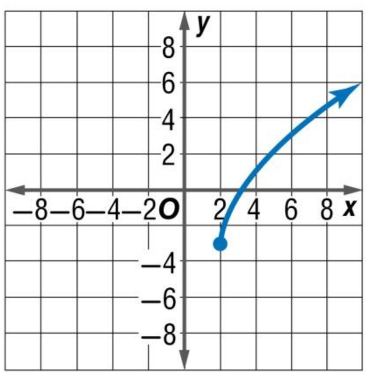
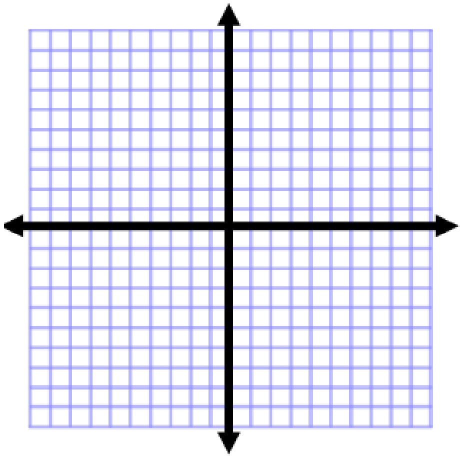


x	y



EXAMPLE 2 Graph Square Root Functions

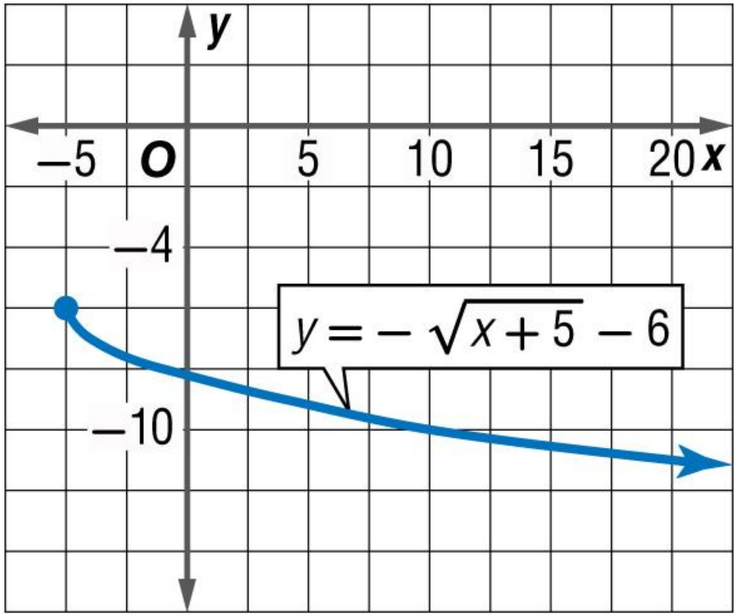
B. Graph the function $y = -\sqrt{x+5} - 6$. State the domain and range.

The minimum point is at $(h, k) = (-5, -6)$. Make a table of values for $x \geq -5$ and graph the function. The graph is the same shape as $f(x) = \sqrt{x}$, but because a is negative, the graph is reflected in the line $f(x) = -6$. It is also translated 5 units left and 6 units down.

EXAMPLE 2 Graph Square Root Functions

Notice the end behavior; as *x* increases, *y* decreases.

Answer: The domain is $\{x \mid x \geq -5\}$ and the range is $\{y \mid y \leq -6\}$.



x	y
–5	–6
–4	–7
–1	–8
4	–9
11	–10

EXAMPLE 2 Check Your Progress

CheckPoint

B. State the domain and range of the function $y = -\sqrt{x+1} - 4$.

A. D: {x | $x \geq -1$}; R: {y | $y \leq -4$}

B. D: {x | $x \geq 1$}; R: {y | $y \geq -4$}

C. D: {x | $x \geq -1$}; R: {y | $y \leq 4$}

D. D: {x | $x \geq 1$}; R: {y | $y \leq 4$}

Real-World Example 3 Use Graphs to Analyze Square Root Functions

A. PHYSICS When an object is spinning in a circular path of radius 2 meters with velocity *v*, in meters per second, the centripetal acceleration *a*, in meters per second squared, is directed toward the center of the circle. The velocity *v* and acceleration *a* of the object are related by the function $v = \sqrt{2a}$.

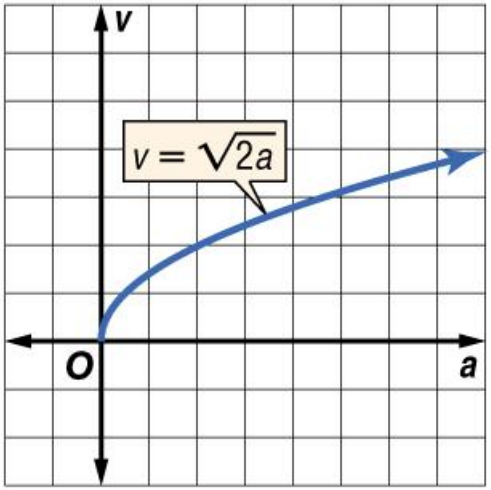
Graph the function in the domain $\{a | a \leq 0\}$.

Real-World Example 3 Use Graphs to Analyze Square Root Functions

The function is $v = \sqrt{2a}$. Make a table of values for $\{a \mid a \le 0\}$ and graph.

Answer:

a	v
0	0
1	1.41
2	2
3	2.45
4	2.83
5	3.16



Real-World Example 3 Use Graphs to Analyze Square Root Functions

B. What would be the centripetal acceleration of an object spinning along the circular path with a velocity of 4 meters per second?

It appears from the graph that the acceleration would be 8 meters per second squared. Check this estimate.

$v = \sqrt{2a}$	Original equation
$4 = \sqrt{2a}$	Replace *v* with 4.
$16 = 2a$	Square each side.
$8 = a$	Divide each side by 2.

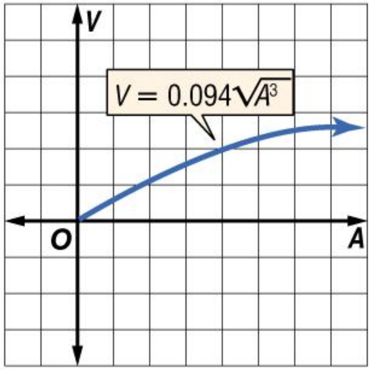
Answer: The centripetal acceleration would be 8 meters per second squared.

Real-World Example 3 — Check Your Progress

CheckPoint

A. GEOMETRY The volume V and surface area A of a soap bubble are related by the function $V = 0.094\sqrt{A^3}$. Which is the graph of this function?

A.



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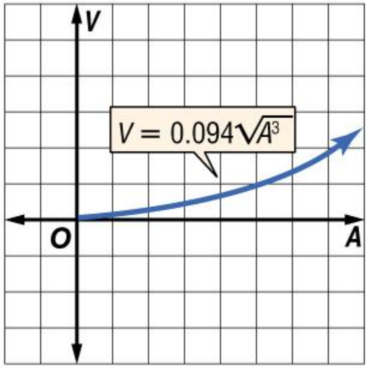

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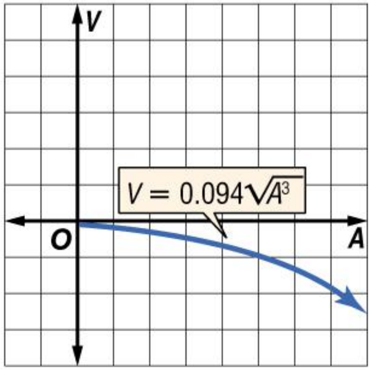

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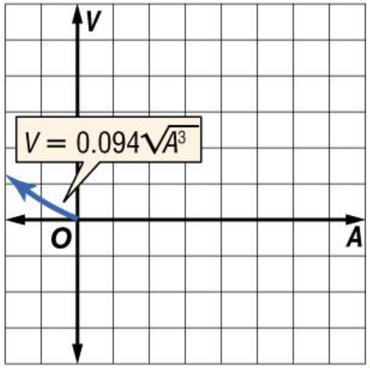
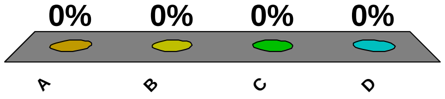




Real-World Example 3 Check Your Progress

CheckPoint

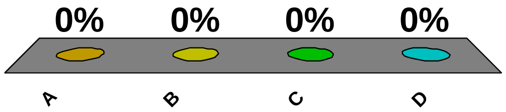
B. GEOMETRY The volume V and surface area A of a soap bubble are related by the function $V = 0.094\sqrt{A^3}$. What would the surface area be if the volume was 3 cubic units?

A. 10.1 units^2

B. 31.6 units^2

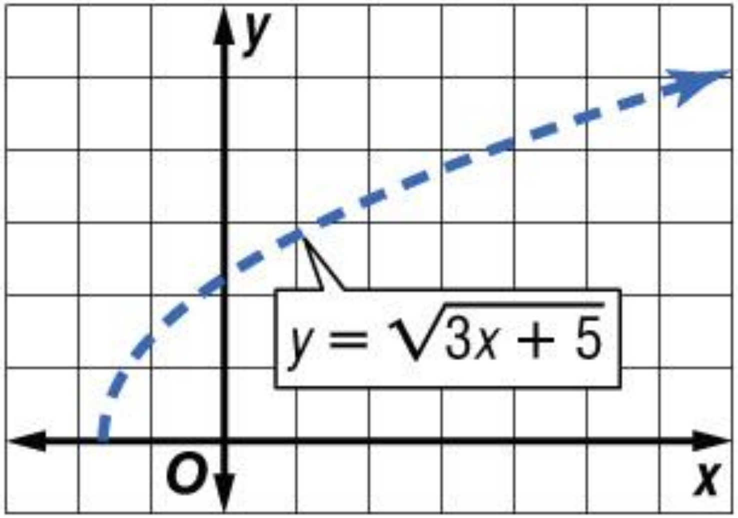
C. 100 units^2

D. 1000 units^2



EXAMPLE 4 Graph a Square Root Inequality

Graph $y > \sqrt{3x+5}$.



Graph the boundary $y = \sqrt{3x+5}$. Since the boundary should not be included, the graph should be dashed.

EXAMPLE 4 Graph a Square Root Inequality

The domain is $x \geq -\frac{5}{3}$. Because *y* is *greater than*, the shaded region should be *above* the boundary and within the domain.

Select a point to see if it is in the shaded region.

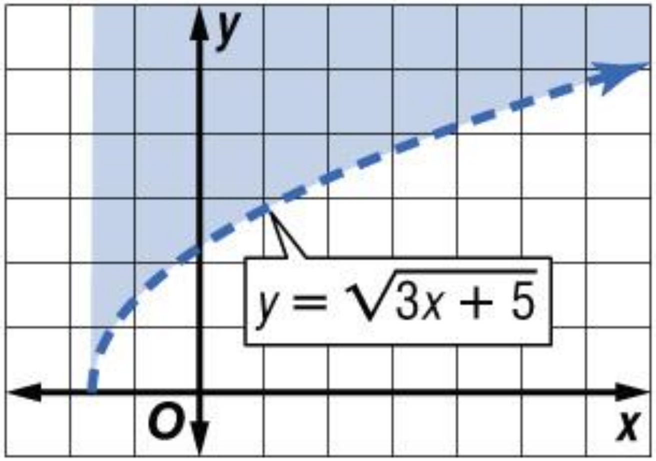
Test (0, 0).

$0 > \sqrt{3(0)+5}$

$0 > \sqrt{5}$

Shade the region that does not include (0, 0).

Answer:

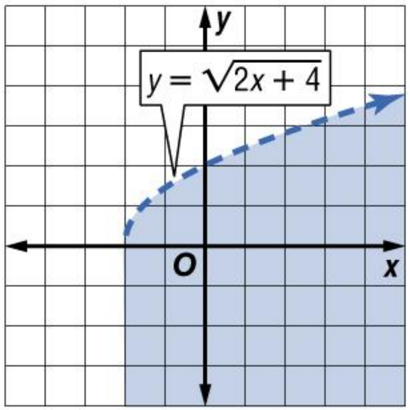


EXAMPLE 4 Check Your Progress

CheckPoint

Which is the graph of $y > \sqrt{2x+4}$?

A.



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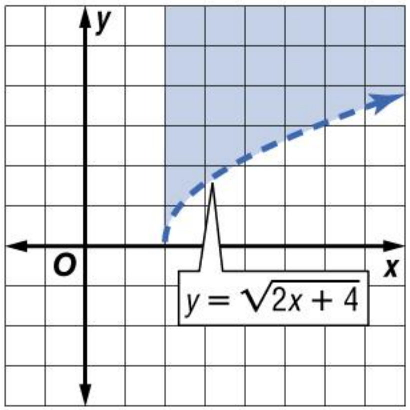

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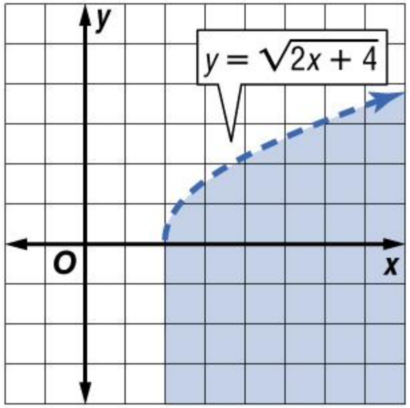

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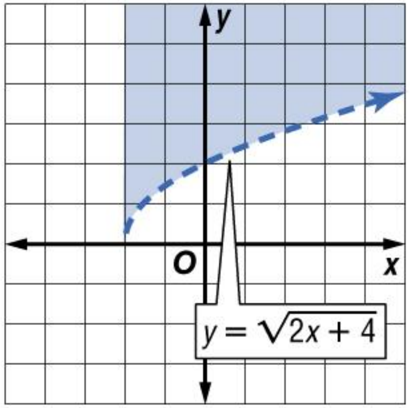

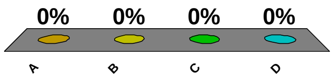

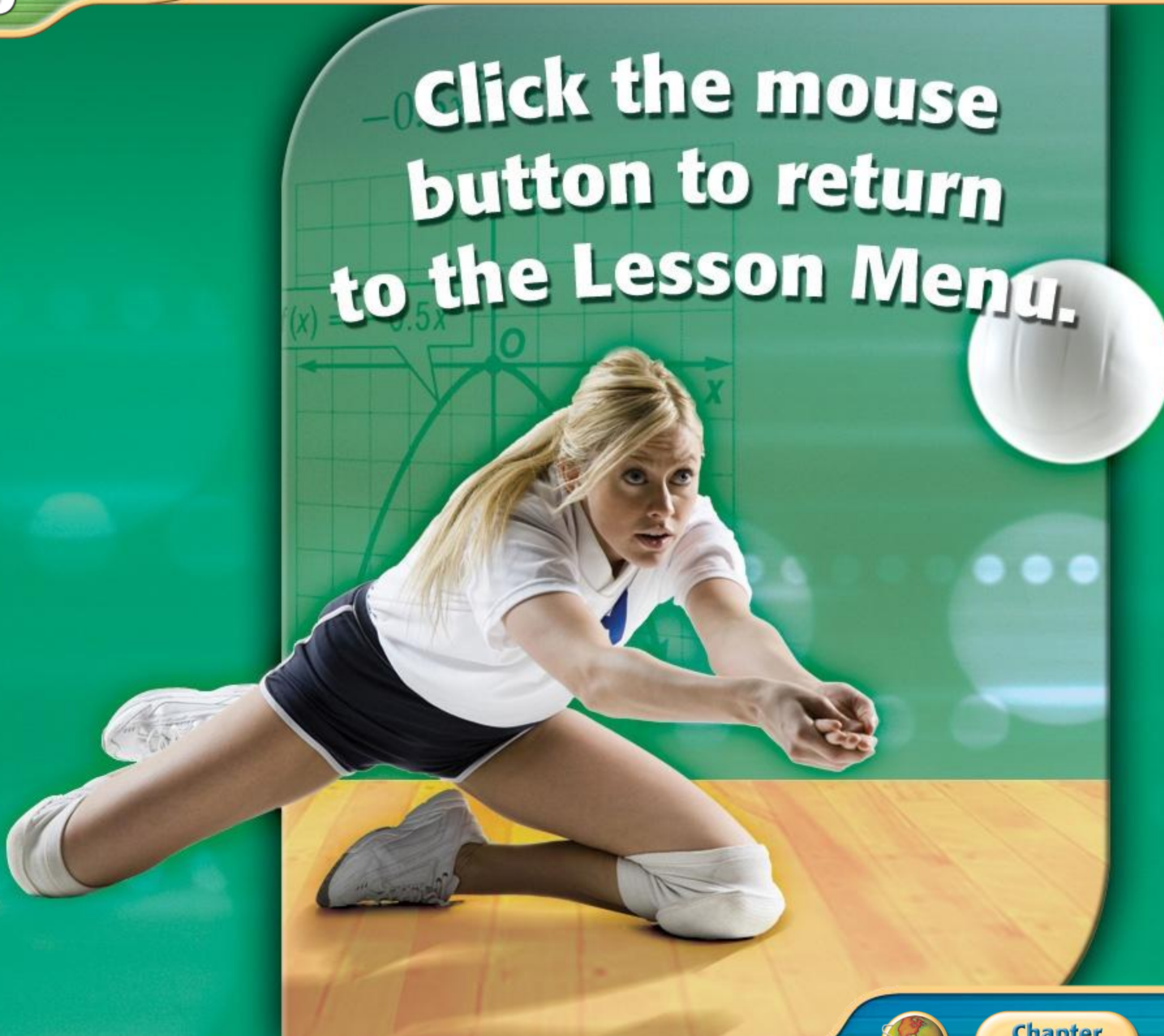
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